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TENSILE FRACTURE OF NOTCHED COMPOSITE LAMINATES

by

C. Poon

Institute for Aerospace Research

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TENSILE FRACTURE OF NOTCHED COMPOSITE LAMINATES

RUPTURE PAR TRACTION DE MATÉRIAUX COMPOSITES STRATIFIÉS

by/par

C. Poon

Institute for Aerospace Research

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ABSTRACT

This report describes the determination of empirical parameters for a fracture model which can be used to predict the uniaxial tensile strength of composite laminates containing notches of various sizes. The parameters required are the characteristic dimension and the unnotched tensile strength of the subject laminate. A data base including notched and unnotched strengths of the subject laminates and elastic ply properties was generated for the determination of these parameters using a statistical procedure based on the two parameter Weibull distribution. Three second generation high toughness graphite fibre reinforced composites, IM6/5245C, IM6/F584 and IM6/1806, and two first generation brittle graphite/epoxy composites, AS4/3501-6 and T300/5208, were used to fabricate the specimens containing either a through-the-thickness cylindrical hole or a countersunk hole.

The two parameter fracture model assessed was found to be capable of predicting accurately the tensile notched strength of composite laminates. The characteristic dimension was found to be dependent on a variety of intrinsic and extrinsic factors; therefore, it must be determined by experiment for each new composite material, laminate configuration, etc. Results showed that a decrease in the characteristic dimension indicated an increase in notch sensitivity. Since the characteristic dimensions of the three second generation composites were found to be larger than those of the first generation composites, they were less notch sensitive. The notched strength of a countersunk hole was predicted by approximating the countersunk hole by a uniform hole with a diameter equal to the outer diameter of the countersunk hole.

RÉSUMÉ

Le présent rapport décrit la détermination de paramètres empiriques en vue de l'élaboration d'un modèle de rupture permettant de prédire la résistance à la traction uniaxiale de matériaux composites stratifiés comportant des entailles de différentes tailles. Les paramètres requis sont la dimension caractéristique et la résistance à la traction en l'absence d'entailles du matériau stratifié. Une base de données contenant les résistances des matériaux stratifiés concernés avec et sans entailles ainsi que les propriétés élastiques des plis a été établie; elle permet de déterminer ces paramètres à l'aide d'une méthode statistique basée sur la distribution de Weibull des deux paramètres. Trois matériaux composites renforcés de fibres de graphite très résistants aux chocs, de deuxième génération, IM6/5245C, IM6/F584 et IM6/1806, et deux matériaux composites graphite/résine époxy cassants de premières génération, AS4/3501-6 et T300/5208, ont été utilisés pour la fabrication d'éprouvettes contenant soit un trou cylindrique traversant toute l'épaisseur, soit une fraisure.

Le modéle de rupture à deux paramètres évalué s'est avéré capable de prédire avec précision la résistance à la traction de matériaux composites stratifiés comportant des entailles. On a constaté que la dimension caractéristique dépendait de divers facteurs intrinsèques et extrinsèques; elle doit donc être déterminée expérimentalement pour chaque nouveau matériau composite, chaque type de stratification, etc. Les résultats montrent qu'une réduction de la dimension caractéristique s'accompagne d'une augmentation de la sensibilité à l'effet d'entaille. Étant donné que les dimensions caractéristiques des trois matériaux composites de deuxième génération étaient supérieures à celles des trois matériaux composites de première génération, les matériaux de deuxième génération étaient moins sensibles à l'effet d'entaille. Le résistance à la traction des matériaux comportant une fraisure était calculée en assimilant la fraisure à un trou uniforme de diamètre égal au diamètre extérieur de la fraisure.

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LIST OF ABBREVIATIONS

a_o	characteristic dimension
A_{ij}	in-plane laminate stiffness
D	hole diameter
E_{T1}, E_{T2}	moduli of elasticity of lamina in 0° and 90° directions
E_x, E_y	effective moduli of elasticity of laminate in x and y directions
G_{12}	shear modulus of lamina
G_{xy}	effective shear modulus of laminate
K_t	orthotropic stress concentration factor for finite width laminate
K_t^{∞}	orthotropic stress concentration factor for infinite width laminate
i	rank number
L	laminate length
R	hole radius
W	laminate width
X_{T1}	average tensile strength of lamina in 0° direction
X_{T2}	average tensile strength of lamina in 90° direction
X_{C1}	average compressive strength of lamina in 0° direction
X_{C2}	average compressive strength of lamina in 90° direction
X_p	abscissa of the Weibull probability plot
Y_p	ordinate of the Weibull probability plot
β_o	Weibull unnotched characteristic strength parameter of laminate
β_N^{∞}	Weibull notched characteristic strength parameter of infinite width laminate

σ_N	notched strength of finite width laminate
σ_N^∞	notched strength of infinite width laminate
σ_o	unnotched tensile strength
σ_y	normal stress
τ_{12}	inplane shear strength of lamina

1.0 INTRODUCTION

In recent years, extensive research and development has been conducted with the aim of increasing the application of composite materials in aircraft, particularly in the primary airframe structures. A major area of study involves the notch sensitivity of composite laminates. The results of these investigations have produced a variety of analytical techniques ranging from comprehensive numerical methods to simplified fracture models that are capable of addressing the complexity of failure processes and damage progression in notched composite laminates.

Several fracture models based on linear elastic fracture mechanics (LEFM) have been proposed extending their applications from metals to composites containing notches [1-3]. Wu [2] indicated that LEFM can only be applied directly to composites when very specific conditions are satisfied. The application of LEFM has been restricted because in the majority of cases no self-similar damage growth is observed. The complexity of the damage mechanism at the notch also raises serious questions as to the applicability of LEFM to composites. On the microscopic level the failure modes occurring within the damage zone at the notch appear in the form of fiber pull-out, matrix micro-cracking and fiber-matrix interfacial failure. On the macroscopic level the major failure modes are delamination, matrix cracking along the fiber direction and fiber breakage.

It is very difficult to determine accurately the stress distribution and to account for the variety of failure modes at the notch. As a result semi-empirical fracture models [4-7], which ignore the physical state of the damage zone, have been developed. These fracture models are based on elastic stress or strain distribution at a distance from the notch and require the use of experimentally determined parameters for the prediction of the notched strength of composites. The fracture behaviour of notched composite laminates has been shown to be affected by many intrinsic and extrinsic variables. Intrinsic variables such as laminate configuration, stacking sequence, constituent properties, fiber volume fraction, fiber-matrix interface characteristics and fabrication procedure, and extrinsic variables such as specimen geometry, notch configuration, moisture content, temperature and loading rate are known to have significant influence on the notched strength and the failure mode of composite laminates [8-21]. Therefore, the effect of these variables on the parameters of the fracture model must be empirically determined.

Numerous experimental investigations have been conducted in the past fifteen years with the aim of deriving the parameters required by the fracture models to account for the effect of the varous intrinsic and extrinsic variables. The majority of the investigations dealt with the notched strength of first generation graphite/epoxy composites subjected to quasi-static uniaxial tensile loading in ambient temperature and moisture conditions. Very limited data are available in the literature for boron/aluminium, graphite/polyimide and second generation high toughness composites.

The objectives of this investigation were to establish a data base for assessing the tensile notch sensitivity of several second generation high toughness composites and to establish the parameters of a fracture model for the prediction of notched strength of composite laminates containing a through-the-thickness hole. Both countersunk and noncountersunk holes were investigated. The notch sensitivity due to an open hole and impact damage under compressive loading are reported elsewhere [22,23]. A widely used two-parameter fracture model developed by Whitney and Nuismer [4] was adopted in this work. For the sake of completeness the theory of this model is outlined in Section 2. Experimental procedures including testing techniques and statistical reduction of data to derive the parameters for notched strength prediction are presented in Section 3. The assessment of the fracture model by using it to perform notched strength predictions is given in Section 4. Conclusions are provided in Section 5.

2.0 NOTCHED STRENGTH MODEL

The fracture model developed by Whitney and Nuismer[4] was used to predict the tensile notched strength of composite laminates containing a through-the-thickness hole. The model is based on the average stress criterion which hypothesizes that failure will occur when the stress at a characteristic dimension, a_o, from the edge of the hole reaches the unnotched strength, σ_o, of the laminate (Fig. 1). This criterion can be expressed by:

$$\frac{1}{a_o}\int_R^{R+a_o} \sigma_y(x,0)dx = \sigma_o \qquad (1)$$

The model requires the knowledge of σ_o, a_o, and the normal stress, σ_y, at the hole in order to predict the notched strength. It is assumed that a_o is a material parameter and as a result it is independent of the size of the hole. Therefore, this fracture model requires only two parameters, σ_o and a_o, for the prediction of tensile strength of composite laminates containing notches of various sizes.

The σ_o of the laminate can be determined by performing a tensile test on the subject laminate or by analysis using lamination plate theory and material allowables. The characteristic dimension must be determined by corroborating with experimental data. In this investigation, both the unnotched strength and the characteristic dimension were determined experimentally. The experimental procedures used will be described in Section 3.

For an infinite orthotropic plate containing a circular hole of radius R and subjected to an uniform stress, σ^∞, applied parallel to the y-axis at infinity (Fig. 1), the normal

stress, σ_y, along the x-axis in the remaining ligament of the laminate can be expressed approximately by [24]:

$$\sigma_y(x, 0) = \frac{\sigma}{2}\left\{2 + \left(\frac{R}{x}\right)^2 + 3\left(\frac{R}{x}\right)^4 - \left(K_T^{\infty} - 3\right)\left[5\left(\frac{R}{x}\right)^6 - 7\left(\frac{R}{x}\right)^8\right]\right\} \quad |x| > R \quad (2)$$

where K_T^{∞} is the orthotropic stress concentration factor for an infinite width plate as expressed by [25]:

$$K_T^{\infty} = 1 + \sqrt{\frac{2}{A_{22}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)} \quad (3)$$

where A_{ij} are the in-plane laminate stiffnesses[26] as determined from laminated plate theory.

The exact solution for the in-plane stress distribution in an infinite orthotropic plate containing a circular hole and subjected to remote uniaxial tension has been formulated by Lekhnitskii [27]. However, the exact solution, using a complex variable mapping approach, is rather complicated. In order to facilitate computation, a polynomial approximation of the exact solution is required. Konish and Whitney [24] demonstrated that the polynomial expression given in Equation 2 provides a very good approximation to the exact solution for [0], [±45] and a series of $[0/\pm 45]_s$ T300/5208 graphite/epoxy laminates. For isotropic plates or laminates with quasi-isotropic lay-up ($K_T^{\infty} = 3.0$), the sixth and eighth order terms in Equation 2 can be omitted.

Combining Equations 1 and 2 and carrying out the integration result in the ratio of notched strength to unnotched strength as expressed by:

$$\frac{\sigma_N^{\infty}}{\sigma_o} = \frac{2(1-\xi)}{2 - \xi^2 - \xi^4 + (K_T^{\infty} - 3)(\xi^6 - \xi^8)} \quad (4)$$

where

$$\xi = \frac{R}{R + a_o}$$

The lower and upper limits of Equation 4 are:

(1) *For very large holes*: $\xi \rightarrow 1, \quad \frac{\bar{\sigma}_N}{\sigma_o} \rightarrow \frac{1}{\bar{K}_T}$

(2) *For very small holes*: $\xi \rightarrow 0, \quad \frac{\bar{\sigma}_N}{\sigma_0} \rightarrow 1$

A series of notch sensitivity curves can be obtained by plotting the values of $\sigma_N^\infty / \sigma_o$ as a function of R. The notch sensitivity curves demonstrating the effect of a_o on the notched strength of composite laminates containing a through-the-thickness circular hole with $K_T^\infty = 3.0$ and $K_T^\infty = 6.0$ are shown in Fig. 2a and Fig. 2b respectively.

3.0 EXPERIMENTAL PROCEDURES AND RESULTS

The notch sensitivity curves shown in Fig. 2 can be used to predict the tensile notched strength of a wide class of orthotropic composite laminates when σ_o and a_o of the material are known. In this section, the experimental procedures and the data reduction techniques used to determine these two parameters are discussed.

The composite materials used and their unidirectional mechanical properties are given in Table 1. The unidirectional mechanical properties were obtained from a data base which has been developed from the characterization work on advanced composite materials conducted at the Institute for Aerospace Research (formerly known as the National Aeronautical Establishment) [28,29]. These properties were used to calculate the in-plane laminate stiffness terms [30] which were then substituted into Equation 3 to calculate the orthotropic stress concentration factors for the laminates tested in this work. The results are shown in Table 2.

Fabrication of the composite panels, determination of the fiber content using the burn-off process and inspection of the panels using ultrasonic C-scan are decribed in References 31, 32 and 33 respectively.

3.1 Unnotched Tensile Tests

Tensile tests were performed on unnotched specimens fabricated with the Narmco IM6/5245C, a second generation high toughness composite, and from the Hercules

AS4/3501-6, a first generation graphite/epoxy composite. The specimens fabricated with the IM6/5245C were nominally 2.5 cm wide by 30 cm long with a gauge section of 15 cm. The specimens fabricated with the AS4/3501-6 were nominally 2.5 cm wide by 43 cm long with a gauge section of 27 cm. These specimen configurations meet the width and gauge length requirements of the ASTM Standards[34]. About half of the specimens tested were strain gauged. A pair of hydraulic grips was used to hold the specimens in an MTS machine with a 500 kN capacity. A gripping technique[35] involving the use of sandblasted aluminium shims was employed to prevent specimen slippage during load transfer. The specimen was loaded in tension to failure at a cross head displacement rate of 0.25 mm/min. A typical stress vs strain curve for the unnotched tension test is shown in Fig. 3. The results of the unnotched tension tests are summarized in Table 3.

3.2 Notched Tensile Tests

Tensile tests were performed on specimens containing a through-the-thickness hole. Both cylindrical and countersunk holes were investigated. The specimens tested were manufactured from three second generation high tounghness composites, which are the Narmco IM6/5245C, the Hexcel IM6/F584 and the Cyanimide IM6/1806, and from two first generation graphite/epoxy, which are the Narmco T300/5208 and the Hercules AS4/3501-6. A test matrix is provided in Table 4.

Most of the specimens tested were nominally 5 cm wide by 30 cm long with a gauge section of 15 cm between hydraulic grips. The specimens fabricated with the AS4/3501-6 materials were nominally 3.8 cm wide by 45 cm long. The specimens containing a countersunk hole were 5 cm wide by 42.5 cm long. These specimen configurations resulted in length-to-width (L/W) and diameter-to-width (D/W) ratios that satisfy the requirements for finite width correction explained in Sub-section 3.3. All specimens tested were strain gauged. A pair of hydraulic grips with sandblasted aluminium shims was used to hold the specimens in an MTS machine with a capacity of 500 kN. A typical setup of the notched tensile test is shown in Fig. 4. The specimens were loaded in tension to failure at a cross head displacement rate of 0.25 mm/min. A typical stress vs strain curve for a notched tension test is shown in Fig. 5. Table 5 summarizes the results of the notched tension tests.

3.3 Data Reduction

The stress at failure of the unnotched specimens, σ_o, was computed based on the failure load and the average cross sectional area of the specimens. The modulus of the laminate was computed by dividing the stress with the strain when the strain was .004 mm/mm. The stress at failure of the notched specimen, σ_N, was computed by dividing the failure load with the average gross cross sectional area of the specimens.

Since the σ_N computed is for specimens with a finite width, a correction factor is required to obtain the notched strength of a laminate with an infinite width, σ_N^{∞}, for use in the fracture model (equation 4). When the material tested is orthotropic, an appropriate

orthotropic finite width correction factor should be used. The correction factor can be determined accurately using analytical methods involving finite element [36], boundary collocation [37], or boundary integral equation [38]. It should be noted that no closed form solutions are available and numerical solutions are required to obtain the orthotropic correction factor. In order to simplify the calculation, an isotropic finite width correction factor[39], shown in Equation 6, was used in this work.

$$\frac{\sigma_N^{\infty}}{\sigma_N} = \frac{K_T}{K_T^{\infty}} = \frac{2 + \left(1 - 2\frac{R}{W}\right)^3}{3\left(1 - 2\frac{R}{W}\right)} \qquad (6)$$

The validity of employing the isotropic width correction factor with orthotropic materials has been addressed extensively in the literature. The use of the isotropic factor instead of the orthotropic factor has been shown to be valid when the D/W ratio is less than or equal to 1/4[40] and the L/W ratio is larger than 3[41]. As mentioned earlier all specimens used in this work meet these two requirements. Using Equation 6, finite width corrections were performed on the experimentally determined notched strengths and the results are shown in Table 5.

3.4 Determination of Parameters for the Fracture Model

The fracture model used in this investigation has two unknown parameters, σ_o and a_o, which must be determined before it can be used to predict the strength of notched laminates. A statistical procedure based on the two-parameter Weibull distribution was used to determine these two parameters for the Narmco IM6/5245C and the Hercules AS4/3501-6. The probability density function for the Weibull distribution [42] is expressed by:

$$f(\sigma_f) = \left[\frac{k(\sigma_f - \phi)^{k-1}}{\beta^k}\right] \exp\left[\frac{(\phi - \sigma_f)}{\beta}\right]^k \qquad (7)$$

where σ_f = stress at failure
β = characteristic strength parameter
k = slope parameter
ϕ = location parameter

When $\phi = 0$, Equation 7 reduces to the two-parameter Weibull distribution. The characteristic strength parameter occurs at the 63.2% failure point for the sample population. The Weibull distribution is a general function which encompasses several well known distributions. For example, the function given in Equation 7 is a simple exponential distribution function when k = 1; the Rayleigh distribution function when k = 2; and a good approximation of the Normal distribution function when k = 3.57, that is, when the mean

and the median values are equal.

3.4.1 Statistical analysis of unnotched strengths

The cumulative form of the probability density function shown in Equation 7 was used to determine the two Weibull parameters, k_o and β_o, for the unnotched strength data shown in Table 3. This cumulative function is expressed by:

$$P(\sigma_o) = 1 - \exp\left[- \left(\frac{\sigma_o}{\beta_o} \right)^{k_o} \right] \tag{8}$$

The unnotched strength data were first ranked from minimium to maximum strength. The stress at failure of each unnotched specimen was then assigned a rank number, i, from 1 through n, where n is the sample size. In this report, $P(\sigma_o)$ is estimated by[43]:

$$P(\sigma_o) = \frac{i - 0.5}{n} \tag{9}$$

The unknown parameters, β_o and k_o, were determined using a plotting technique. In the Weibull probability plot, the abscissa and the ordinate are shown in Equations 10 and 11 respectively.

$$X_p = \ln\left[\ln\left(\frac{1}{1 - P(\sigma_o)} \right) \right] \tag{10}$$

$$Y_p = \ln[\sigma_o] \tag{11}$$

The Weibull plots of the unnotched strength data shown in Table 3 are shown in Figs. 6 and 7. Using linear regression, a straight line was fitted through the data. The parameter k_o was obtained by determining the slope of the straight line. The parameter β_o was determined by reading off the strength value corresponding to the intersection of the fitted line and a horizontal line corresponding to $P(\sigma_o) = 63.2\%$. The results of the calculation are summarized in Table 6.

3.4.2 Statistical analysis of notched strengths

The notched strength data shown in Table 5 were analysed using the same procedure discussed in Sub-section 3.4.1 for the Narmco IM6/5245C and the Hercules AS4/3501-6 materials. The Weibull plots of the notched strength data are shown in Figs. 8 and 9. The parameters, β_N^{∞} and k_N, for the two materials are shown in Table 7.

3.4.3 Determination of a_o

The characteristic strength parameters, β_o and β_N^{∞}, were substituted into Equation 4 to yield Equation 12 which was solved iteratively for a_o. The results are shown in Table 8.

$$\frac{\beta_N^{\infty}}{\beta_o} = \frac{2\,(1-\xi)}{2 - \xi^2 - \xi^4 + (K_T^{\infty} - 3)(\xi^6 - \xi^8)} \qquad (12)$$

4.0 ASSESSMENT OF FRACTURE MODEL

As mentioned earlier, the notch sensitivity of composite laminates is affected by many intrinsic and extrinsic factors. The applicability of the fracture model would be greatly enhanced if the parameters of the model could be applied universally to predict the notched strength of laminates manufactured with different composite materials and laminate configurations and containing different notch shapes. A major requirement for the fracture model to be universal is that the characteristic dimension is a material constant and is independent of specimen geometry and laminate configuration.

Whitney and Nuismer [44] claimed and substantiated with experimental evidence that using the average stress failure criterion with a constant value of a_o = 2.3 mm resulted in acceptable predictions for the fracture strength of a number of different laminates of an AS/3501-5 graphite epoxy material system. Their results also showed that using the model resulted in good strength predictions for countersunk and non-countersunk holes, and for normal and slant cracks.

Several investigators attempted to use the same fracture model that Whitney and Nuismer used for predicting the notched strength of different laminates and in several cases demonstrated poor agreement between predicted and experimental results. Karlak [5] performed fracture tests on a series of quasi-isotropic laminates with a hole of different diameters and showed that the characteristic dimension was proportional to the square root of the hole diameter. Whitney and Kim [9] found that stacking sequence has a significant effect on the notch sensitivity of composite laminates. Pipes, et al [45] proposed a fracture model based on the principle of superposition of notched strength data for several composite materials and laminate stacking sequences. Through the development of a notched strength-radius superposition method and the definition of two radius shift parameters, a single master curve was developed which allows the prediction of the influence of laminate stacking sequence upon the notched strength of different composite materials. However, this approach requires additional tests to determine the additional parameters and as a result the process is more complicated and expensive. The accuracy of

prediction is only marginally improved for those cases where the characteristic dimension increases with notch size.

A comprehensive study [46] was conducted with the objective of collecting a massive set of experimental results and comparing the data with predicted results generated by several commonly used fracture models. A total exceeding 2800 notched strength data was collected and filed in a computer data bank for graphite/epoxy, boron/aluminium and graphite polyimide laminates. The study pointed out that the characteristic dimensions for different material systems are in fact independent of notch size in most cases or exhibit only a slight increase with notch size. The study also indicated that the characteristic dimension is strongly influenced by material lay-up patterns. For example, the values of the characteristic dimension of AS1/3501-6 vary from 199.6 mm for the $[0]_6$ lay-up to -0.06 mm for the $[90]_4$ lay-up. The negative value of the characteristic dimension, which has no physical meaning, is caused by the condition that $\sigma^{\infty}{}_N/\sigma_o < 1/K^{\infty}{}_T$. This condition is below the lower limit of Equation 4 and therefore, in theory, this fracture model cannot be used to predict the notched strength of laminates with the $[90]_4$ lay-up. However, as a result of the curve-fitting procedure employed, a negative value was obtained which gave the best correlation with the test data.

Reviewing the summary of the results given in Reference 46, it is observed that the effect of other intrinsic material variables on the notch sensitivity is manifested by the significant differences among the values of the characteristic dimensions for identical lay-ups obtained from different sources, suggesting significant influence of batch variation of constituent properties, fiber volume fraction, fabrication procedures, etc. For example, Lagace[47] obtained a value of 1.4 mm for the characteristic dimension of AS1/3501-6 with a lay-up of $[0/\pm 45]_s$ which differs significantly from the value of 3.37 mm obtained by Cruse[38] under identical test conditions.

In this work the effects of several intrinsic factors were controlled by adopting a consistent autoclave fabrication technique and determining accurately the fiber volume content. The quality of the laminates was assured by using ultrasonic C-scan inspection. The inherent scatter of the test data was accounted for by using adequate sample sizes and the Weibull statistics were used to obtain an estimate of the characteristic dimension. The established fracture model was assessed by using it to predict the notched strength of (a) IM6/5245C laminates containing a hole, two different diameters were tested; (b) IM6/5245C laminates containing a countersunk hole; (c) laminates fabricated with different second generation high toughness composites; and (d) laminates fabricated with a first generation composite, T300/5208.

4.1 Notched Strength Prediction

A comparison between the predicted values of $\sigma_N^{\infty}/\sigma_o$ and the experimental results generated for IM6/5245C laminates is shown in Fig. 10. The predicted values of $\sigma_N^{\infty}/\sigma_o$ were generated using Equation 4 for a range of hole sizes. The a_o = 3.43 mm established

earlier in Section 3 was used for the prediction. This a_o was obtained by corroborating with test results for quasi-isotropic laminates containing a single circular hole. Two hole diameters, 6.35 mm and 9.53 mm, were tested for this purpose. It is noted that the predicted values agree, as expected, very well with the experimental results for these two hole sizes. However, the values of σ_N^∞/σ_o for the hole diameters, 3.18 mm and 12.7 mm, are predicted unconservatively. The discrepancy between prediction and experimental results appears to be related to the small scatter of test data resulting from small sample sizes for these two hole diameters. The number of tests performed for each of these two diameters is three which is much smaller than the eleven tests performed for the two intermediate hole diameters. Although confirmation is not possible without additional test results, it appears that the small scatter may not be representative of the true population scatter which leads to a poor correlation with the experimental results.

The fracture model with a_o = 3.43 mm predicts the values of σ_N^∞/σ_o for the countersunk hole very well when the countersunk hole is approximated by a uniform through-the-thickness hole with a diameter equal to the outer diameter of the countersunk hole. This finding agrees with that obtained by Nuismer and Labor [44].

The results for predicting the values of σ_N^∞/σ_o for the two second generation high toughness composites, IM6/1806 and IM6/F584, and for a first generation brittle composite, T300/5208, are summarized in Fig. 11. The a_o = 3.43 mm established from the test results of IM6/5245C is used in the fracture model for predicting the values of σ_N^∞/σ_o for IM6/1806 and IM6/F584. The a_o = 1.48 mm established from the test results of AS4/3501-6 is used for predicting the values of σ_N^∞/σ_o for T300/5208. As shown in Fig. 11, prediction agrees with experimental results only in the case for IM6/F584, a composite material with mechanical properties similar to those of IM6/5245C (Table 1).

Using the procedure discribed in Section 3, the characteristic dimensions for IM6/1806, IM6/F584 and T300/5208 were determined to be 4.9 mm, 3.3 mm and 2.7 mm respectivley. It must be emphasized that these values have been determined based on 3 to 4 tests for each material and they may not be strong estimates of the true a_o for these materials. Using these values the prediction of σ_N^∞/σ_0 was performed over a range of hole sizes for these materials and the results are presented in Fig. 12. It is observed that a decrease in a_o results in an increase in notch sensitivity. Therefore a ranking of the notch sensitivity of composite materials based on a_o is possible.

Although the a_o of a composite material can be correlated to notch sensitivity, it cannot indicate the change in failure mode resulting from a change in notch sensitivity. As shown in Fig. 13 an increase in notch sensitivity resulting from an increase in hole diameter from 3.18 mm to 12.7 mm corresponds to a change in failure mode from slant fracture to flat fracture. For intermediate hole sizes, mixed modes involving slant and flat fractures predominate. As shown previously a constant value of a_o = 3.43 mm can be used to predict, with mixed results, the notched strength for all these hole sizes. The countersinking of a hole with a nominal diameter = 6.71 mm results in a significant increase in notch sensitivity and

a flat fracture mode as shown in Fig. 14. The same value of a_o = 3.43 mm can be used to obtain a good correlation with the test results by approximating the countersunk hole by a uniform through-the-thickness hole with a diameter equal to the outer countersink diameter.

5.0 CONCLUSIONS

An investigation was conducted which led to the determination of the two parameters of a fracture model for the prediction of the tensile notched strength of several graphite fibre reinforced composites. Key conclusions drawn from this work are:

1. The merit of the two parameter fracture model based on the average stress criterion is the inherent simplicity of the model. It was found to be capable of predicting the tensile strength of composite laminates containing notches of various sizes.

2. The two parameters, which are the unnotched strength of the subject laminate and the characteristic dimension of the material, must be determined before the model can be used for prediction.

3. The determination of the characteristic dimension requires a data base including the notched strength and elastic ply properties of the subject laminate. Since the characteristic dimension is dependent on a variety of intrinsic and extrinsic factors, it must be determined by experiment for each new composite material, laminate configuration, etc.

4. A statistical procedure based on the two parameter Weibull distribution was used to obtain an estimate of the characteristic strength parameters for the notched and unnotched strengths of the subject laminate. The ratio of the characteristic strength parameters was used to calculate the characteristic dimension.

5. The characteristic dimension can be used as an indicator for the notch sensitivity of different composite materials. A decrease in the characteristic dimension was found to indicate an increase in notch sensitivity.

6. The second generation high toughness composites used in this study which included the IM6/5245C, IM6/F584 and IM6/1806 were significantly less notch sensitive than the first generation composite T300/5208. The characteristic dimensions of these composite materials were larger than that of the T300/5208.

7. Approximating the countersunk hole by a uniform through-the-thickness hole with a diameter equal to the outer diameter of the countersunk hole allows the fracture model to be used for predicting the tensile notched strength of a countersunk hole.

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7.0 REFERENCES

1. Waddoups, M. E., Eisenmann, J. R. and Kaminski, B. E., "Macroscopic Fracture Mechanics of Advanced Composite Materials," Journal of Composite Materials, Vol. 5, 1971, pp. 446-454.

2. Wu, E. M., "Fracture Mechanics of Anisotropic Plates," in Composite Materials Workshop, Tsai, S. W., Halpin, J. C. and Pagano, N. J., Eds., Technomic Pub. Co., Inc., Lancaster, PA, 1968, pp. 20-43.

3. Mar, J. W. and Lin, K. Y., "Fracture Mechanics Correlation for Tensile Failure of Filamentary Composites with Holes," Journal of Aircraft, Vol. 14, No. 7, July 1977, pp. 703-704.

4. Whitney, J. M. and Nuismer, R. J., "Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations," Journal of Composite Materials, Vol. 8, 1974, pp. 253-265.

5. Karlak, R. F., "Hole Effects in a Related Series of Symmetrical Laminates," Proceedings of Failure Modes in Composites, The Metallurgical Society of AIME, Chicago, 1977, pp. 105-117.

6. Pipes, R. B., Wetherhold, R. C. and Gillespie, J. W., Jr., "Notched Strength of Composite Materials," Journal of Composite Materials, Vol. 12, 1979, pp. 148-160.

7. Poe, C. C., Jr., "A Unified Strain Criterion for Fracture of Fibrous Composite Laminates," Engineering Fracture Mechanics, Vol. 17, No. 2, 1983, pp. 153-171.

8. Poe, C. C., Jr., "Fracture Toughness of Boron/Aluminium Laminates with Various Proportions of 0° and ±45° Plies," NASA Technical Paper 1707, November, 1980.

9. Whitney, J. M. and Kim, R. Y., "Effect of Stacking Sequence on the Notched Strength of Composite Laminates," U. S. Air Force Materials Laboratory Technical Report, AFML-TR-76-177, November, 1977.

10. Daniel, I. M., "Biaxial Testing of Graphite/Epoxy Composites Containing Stress Concentrations," Air Force Materials Laboratory Technical Report, AFML-TR-76-244-PT-2, June 1977.

11. Garcia, J. I., "Static Tensile Behaviour of $[0]_6$, $[0/\pm 5]_s$, $[0/\pm 10]_s$, and $[0/\pm 15]_s$ Graphite/Epoxy Laminates with Holes," Technology Laboratory for Advanced Composites, M.I.T. Department of Aeronautics and Astronautics, TELAC Report 80-7, Cambridge, Massachusetts, June 1980.

12. Garcia, J. I., "Fracture of $[\pm 5/0]_s$, $[\pm 10/0]_s$, and $[\pm 15/0]_s$ Graphite/Epoxy Coupons under Tension," Technology Laboratory for Advanced Composites, M.I.T. Department of Aeronautics and Astronautics, TELAC Report 80-8, Cambridge, Massachusetts, August 1980.

13. McManus, L. H., "Failure Modes in a Family of Graphite/Epoxy Laminates," Technology Laboratory for Advanced Composites, M.I.T. Department of Aeronautics and Astronautics, TELAC Report 81-8, Cambridge, Massachusetts, May 1981.

14. Whitney, J. M. and Kim, R. Y., "High Temperature Tensile Strength of Graphite/Epoxy Laminates Containing Circular Holes," Journal of Composite Materials, Vol. 10, 1976, pp.319-324.

15. Kennedy, J. M., "Fracture Behaviour of Hybrid Composite Laminates," AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, AIAA Paper No. 83-0804-CP, Lake Tahoe, Nevada, May 1983.

16. Harris, C. E. and Morris, D. H., "A Characterization of the Fracture Behaviour of Thick, Notched, Laminated Graphite/Epoxy Composites," VPISU Report No. VPI-E-83-31, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 1983.

17. Ochiai, S. and Peters, P. W. M., "Effect of Notch Length, Specimen Thickness, Ply Thickness and Type of Defect on the Fracture Behaviour of Angle-Ply Graphite/Epoxy Composites," Journal of Materials Science, Vol.17, 1982, pp. 2324-2336.

18. Raizenne, M. D. and Hall, S. R., "The Effect of Through-the-Thickness Holes on the Tensile Strength of Thick Laminates," Proceedings of the Second International Conference on Analytical and Testing Methodologies for Design with Advanced Materials (ATMAM '89), Montreal, Quebec, Canada, August 16-18, 1989.

19. Prewo, K. M., "The Effect of Ply Lay-Up Sequence on the Fracture Toughness of Boron Aluminium," Journal of Composite Materials, Vol, 12, 1978, pp. 40-52.

20. Garber, D. P., Morris, D. H. and Everrett, R. A., Jr., "Elastic Properties and Fracture Behaviour of Graphite/Polyimide Composites at Extreme Temperatures," Composites for Extreme Environment, ASTM STP 768, American Society for Testing and Materials, Philadelphia, Pa., 1982, pp. 73-91.

21. Petker, I. and Stern, B. A., "Microstructure, the Interface, and the Structural Performance to 700°F of Graphite/Polyimide Composite," Proceedings of the 23rd National SAMPE Symposium and Exhibition, SAMPE, Vol. 23, 1978, pp. 75-79.

22. Poon, C., Scott, R. F. and Lee, S., "Testing of New-Generation Carbon Fiber/Toughened Resin Epoxy Systems," Polymer Composites, Vol. 9, No. 5, October 1988, pp. 318-329.

23. Poon, C., Benak, T. and Gould, R., "Assessment of Impact Damage in Toughened Resin composites," Theoretical and Applied Fracture Mechanics, Vol. 13, 1990, pp.81-97.

24. Konish, H. J. and Whitney, J. M., "Approximate Stresses in an Orthotropic Plate Containing a Circular Hole," Journal of Composite Materials, Vol. 9, 1975, pp. 157-166.

25. Lekhnitskii, S. G., Anisotropic Plates, Translated from the Second Russian Edition by S. W. Tsai and T. Cheron, Gordon and Breach, Science Publishers, Inc. New York, 1968.

26. Jones, R. M., Mechanics of Composite Materials, Scripta Book Co., Washington, D. C., 1975.

27. Lekhnitskii, S. G., Theory of Elasticity of an Anisotropic Body, Holden-Day, San Francisco, 1963.

28. Lee, S., Scott, R. F., Gaudert, P. C., Ubbink, W. H. and Poon, C., "Mechanical Testing of Toughened Resin Composite Materials," Composites, Vol. 19, No. 4, July 1988, pp. 300-310.

29. Lee, S., Scott, R. F. and MacKenzie, C., "Material Characterization of Hercules AS4/3501-6 Graphite Epoxy," LTR-ST-1515, National Aeronautical Establishment, National Research Council Canada, Ottawa, September 1984.

30. Hall, S. R., "LAMCAL: An Iterative Design Package for Performing a Laminated Plate Theory Analysis of Composite Materials," LTR-ST-1656, National Aeronautical Establishment, National Research Council Canada, Ottawa, November 1987

31. Lee, S., Scott, R. F. and Colton, J., "Methods for Processing Aerospace Composite Materials at NAE," LTR-ST-1558, National Aeronautical Establishment, National Research Council Canada, Ottawa, December, 1985.

32. Gaudert, P. C., Scott, R. F. and Lee, S., "Development of a Burn-Off Test for the Fibre Content of Carbon Fibre Reinforced Epoxy Composites," LTR-ST-1631, National Aeronautical Establishment, National Research Council Canada, Ottawa, June 1987.

33. Fahr, A. and Charlesworth, A. M., "An Introduction to the Ultrasonic C-Scan Inspection of Advanced Composite Materials," LTR-ST-1602, National Aeronautical Establishment, National Research Council Canada, Ottawa, November 1986.

34. "Standard Test Method for Tensile Properties of Fiber-Resin Composites," ASTM Standards D-3039, American Society for Testing and Materials, Vol. 15.03, 1989.

35. Heath, J. B. R., Komorowski, J. P. and Shaver, B. E., "Design and Development of Hydraulically-Clamped Grips for Composite Material Testing," LTR-ST-1729, National Aeronautical Establishment, National Research Council Canada, Ottawa, November 6, 1989.

36. Alturi, S. N., Kobayashi, A. S. and Nakagaki, M., "A Finite Element Program for Fracture Mechanics Analysis of Composite Materials," Fracture Mechanics of Composites, ASTM STP 593, American Society for Testing and Materials, 1975, pp. 86-98.

37. Bowie, O. L. and Freeze, C. E., "Central Crack in Plane Orthotropic Rectangular sheet," International Journal of Fracture Mechanics, Vol. 8, Nol 1, 1972, pp. 49-57.

38. Cruse, T. A. and Osias, J. R., "Exploratory Development on Fracture Mechanics of Composite Materials," U. S. Air Force Materials Laboratory Technical Report, AFML-TR-74-111, April 1974.

39. Peterson, R. E., Stress Concentration Factors, John Wiley & Son, New York, 1974.

40. Gillespie, J. W., Jr. and Carlsson, L. A., "Influence of Finite Width on Notched Laminate Strength Predictions," Composite Science and Technology, Vol. 32, 1988, pp. 15-30.

41. Mar, J., "Fracture and Fatigue in Bi-Materials," Mechanics of Composite Review, Air Force Materials Laboratory and Air Force Office of Scientific Research Technical Report, October 1976, pp. 117-122.

42. Weibull, W., "A Statistical Distribution Function of Wide Applicability," Transactions, American Society for Mechanical Engineers, Journal of Applied Mechanics, Vol. 73, September 1951, pp. 293-297.

43. Tenn, L. F., "Statistical Analysis of Fibrous Composite Strength Data," Test Methods and Design Allowables for Fibrous Composites, ASTM STP 734, Chamis, C. C., Editor, American Society for Testing and Materials, 1981, pp. 229-244.

44. Nuismer, R. J. and Labor, J. D., "Applications of the Average Stress Failure Criterion: Part I -- Tension," Journal of Composite Maaterials, Vol. 12, 1978, pp. 238-249.

45. Pipes, R. B., Wetherhold, R. C. and Gillespie, J. W., Jr., "Noteched Strength of Composite Materials," Journal of Composite Materials, Vol. 13, April 1979, pp. 148-160.

46. Awerbuch, J. and Madhukar, M. S., "Notched Strength of Composite Laminates: Predictions and Experiments -- A Review," Journal of Reinforced Plastics and Composites," Vol. 4, January 1985, pp. 3-158.

47. Lagace, P. A., "Static Tensile Fracture of Graphite/epoxy," Ph. D. Dissertation, Department of Aeronautics and Astronautics, M.I.T., Cambridge, Mass., June 1980.

TABLE 1. MECHANICAL PROPERTIES OF COMPOSITE MATERIALS [28, 29]

(a) Elastic Moduli

Material Name	0° Tensile Modulus (E_{T1}), GPa	90° Tensile Modulus (E_{T2}), GPa	Shear Modulus (G_{12}), GPa	Poisson Ratio (μ_{12})
Narmco IM6/5245C	166.2	8.3	5.5	0.31
Hexcel IM6/F584	171.0	9.0	5.9	0.32
Cyanamid IM6/1806	155.1	10.3	3.9	0.32
Hercules AS4/3501-6	140.0	8.2	6.2	0.30

(b) Ultimate Strengths

Material Name	0° Tensile Strength (X_{T1}), MPa	0° Compressive Strength (X_{C1}), MPa	90° Tensile Strength (X_{T2}), MPa	90° Compressive Strength (X_{C2}), MPa	Inplane Shear Strength (τ_{12}), MPa
Narmco IM6/5245C	2,620.0	1,278.3	60.0	220.6	117.9
Hexcel IM6/F584	2,546.2	1,340.0	47.6	232.4	122.7
Cyanamid IM6/1806	1,840.9	1,178.3	40.7	199.3	91.0
Hercules AS4/3501-6	2,144.3	824.6	46.2	172.4	110.3

TABLE 2. STRESS CONCENTRATION FACTORS FOR COMPOSITE LAMINATES

Material Name	Lay-up	Orthotrophic In-Plane Stiffness Terms*, GPa				Effective Elastic Moduli*, GPa				Stress Concentration Factor $\left(K_T^{\infty}\right)$
		A_{11}	A_{22}	A_{66}	A_{12}	E_y	E_x	G_{yx}	μ_{yx}	
IM6/5245C	I	18.9	18.9	6.6	5.8	59.0	59.0	22.4	0.31	3.0
IM6/F584	I	18.9	18.9	6.6	5.8	61.8	61.8	23.7	0.30	3.0
IM6/1806	I	17.5	17.5	5.9	5.7	54.5	54.5	20.5	0.33	3.0
T300/5208	I	15.5	15.5	5.3	4.9	54.3	54.3	20.9	0.30	3.0
IM6/5245C	II	23.8	9.4	4.6	4.0	97.2	38.2	20.3	0.42	3.6
AS4/3501-6	II	20.7	8.4	4.2	3.3	83.3	33.7	18.1	0.40	3.6

NOTES:

Lay-up I = $[+45/0/-45/90]_{6s}$

Lay-up II = $[+45/0/-45/0/90/0/+45/0/-45/0]_{2s}$

* Calculated using LAMCAL

TABLE 3. UNNOTCHED TENSILE TEST RESULTS

Material Name	Lay-Up*	Specimen Number	Ultimate Tensile Strength (σ_o), MPa	Modulus @ .004 Strain (E_y), GPa
IM6/5245C	I	212-A	846.7	N/A
IM6/5245C	I	212-B	853.6	N/A
IM6/5245C	I	212-C	842.5	N/A
IM6/5245C	I	212-D	766.0	N/A
IM6/5245C	I	212-E	786.7	N/A
IM6/5245C	I	212-F	862.5	N/A
IM6/5245C	I	212-G	787.4	N/A
IM6/5245C	I	212-H	787.4	57.9
IM6/5245C	I	211-X	836.3	N/A
IM6/5245C	I	211-Y	912.2	N/A
IM6/5245C	I	211-Z	815.6	N/A
IM6/5245C	II	157-W	1,274.2	90.3
IM6/5245C	II	157-X	1,370.7	89.6
IM6/5245C	II	157-Y	1,200.4	91.0
IM6/5245C	II	157-Z	1,256.2	88.3
AS4/3501-6	II	233-A	1,150.0	78.6
AS4/3501-6	II	233-B	1,179.0	83.4
AS4/3501-6	II	233-C	1,079.0	83.4
AS4/3501-6	II	233-D	1,170.0	80.7
AS4/3501-6	II	233-E	1,116.3	78.6
AS4/3501-6	II	233-F	1,137.6	80.7
AS4/3501-6	II	233-G	1,155.6	81.4
AS4/3501-6	II	233-H	1,261.1	82.0
AS4/3501-6	II	233-I	1,150.0	79.3
AS4/3501-6	II	233-J	1,163.8	81.4
AS4/3501-6	II	233-K	1,119.7	79.3
AS4/3501-6	II	233-L	1,230.0	N/A

* see Table 2

TABLE 4. TEST MATRIX FOR NOTCHED SPECIMENS

Material Name	Lay-Up*	Hole Type	Diameter mm	Number of Tests
IM6/5245C	I	Circular	3.18	3
IM6/5245C	I	Circular	6.35	11
IM6/5245C	I	Circular	9.53	11
IM6/5245C	I	Circular	12.7	3
IM6/5245C	II	Countersunk 100°	6.71 (min.) 13.46 (max.)	9
IM6/1806	I	Circular	6.35	3
IM6/F584	I	Circular	6.35	4
T300/5208	I	Circular	6.35	3
AS4/3501-6	II	Circular	6.71	11

* see Table 2

TABLE 5. NOTCHED TENSILE TEST RESULTS

Material Name	Lay-up*	Hole Type	Diameter mm	Specimen Number	Modulus @ .004 Strain (E_y), GPa	Ultimate Tensile Strength (σ_N), MPa	Finite Width Correction Factor (k_T/k_T^∞)	Infinite Tensile Strength (σ_N^∞), MPa
IM6/5245C	I	Circular	3.18	H-210-B	60.0	573.3	1.004	575.7
IM6/5245C	I	Circular	3.18	H-210-C	61.4	590.8	1.004	593.2
IM6/5245C	I	Circular	3.18	H-211-D1	60.0	581.1	1.004	583.5
IM6/5245C	I	Circular	6.35	H-211-B	60.7	503.6	1.017	512.2
IM6/5245C	I	Circular	6.35	H-211-C	62.7	524.1	1.017	533.1
IM6/5245C	I	Circular	6.35	H-211-D	60.7	505.7	1.017	514.3
IM6/5245C	I	Circular	6.35	H-211-E	61.4	499.4	1.017	507.9
IM6/5245C	I	Circular	6.35	H-211-F	59.3	486.0	1.017	494.3
IM6/5245C	I	Circular	6.35	H-211-G	61.4	499.9	1.017	508.4
IM6/5245C	I	Circular	6.35	H-211-H	57.9	494.6	1.017	503.1
IM6/5245C	I	Circular	6.35	H-211-I	61.4	498.5	1.017	507.0
IM6/5245C	I	Circular	6.35	H-36-1A	59.3	484.5	1.017	492.8
IM6/5245C	I	Circular	6.35	H-36-2	59.2	482.6	1.017	490.9
IM6/5245C	I	Circular	6.35	H-133	59.0	478.5	1.017	486.7
IM6/5245C	I	Circular	9.53	H-210-G	62.7	429.9	1.041	447.4
IM6/5245C	I	Circular	9.53	H-210-H	N/A	418.5	1.041	435.5
IM6/5245C	I	Circular	9.53	H-210-I	60.7	431.0	1.041	448.5
IM6/5245C	I	Circular	9.53	H-211-A	61.4	441.5	1.041	459.4
IM6/5245C	I	Circular	9.53	H-212-D	N/A	452.9	1.041	471.3
IM6/5245C	I	Circular	9.53	H-212-E	58.6	419.7	1.041	436.7
IM6/5245C	I	Circular	9.53	H-212-F	62.7	440.2	1.041	458.1
IM6/5245C	I	Circular	9.53	H-212-G	62.7	433.3	1.041	450.9
IM6/5245C	I	Circular	9.53	H-212-H	59.3	441.0	1.041	458.9
IM6/5245C	I	Circular	9.53	H-212-I	60.7	424.7	1.041	441.9
IM6/5245C	I	Circular	9.53	H-212-J	62.1	440.6	1.041	458.5

* see Table 2

TABLE 5. (CONTINUED)

Material Name	Lay-up*	Hole Type	Diameter mm	Specimen Number	Modulus @ .004 Strain (E_y), GPa	Ultimate Tensile Strength (σ_N), MPa	Finite Width Correction Factor (k_T/k_T^{∞})	Infinite Tensile Strength (σ_N^{∞}), MPa
IM6/5245C	I	Circular	12.7	H-210-D	61.4	362.2	1.077	390.1
IM6/5245C	I	Circular	12.7	H-210-E	60.0	367.8	1.077	396.1
IM6/5245C	I	Circular	12.7	H-210-F	56.5	370.9	1.077	399.5
IM6/5245C	II	Countersink 100°	6.71 (min) 13.46 (max)	157-1A	89.6	584.9	1.087	635.7
IM6/5245C	II			157-1B	89.6	596.9	1.087	648.8
IM6/5245C	II			157-1D	90.3	552.8	1.087	600.9
IM6/5245C	II			157-1E	90.3	605.0	1.087	657.6
IM6/5245C	II			157-1F	89.6	568.0	1.087	617.4
IM6/5245C	II			157-1G	88.9	545.2	1.087	592.7
IM6/5245C	II			157-1H	89.6	563.2	1.087	612.2
IM6/5245C	II			157-1I	88.3	603.8	1.087	656.3
IM6/5245C	II			157-1J	90.3	623.2	1.087	677.4
IM6/1806	I	Circular	6.35	H-150-A	N/A	403.3	1.017	410.2
IM6/1806	I	Circular	6.35	H-150-B	54.5	390.9	1.017	397.6
IM6/1806	I	Circular	6.35	H-150-D	55.3	404.0	1.017	410.9
IM6/F584	I	Circular	6.35	H-91-D	62.1	513.1	1.017	521.9
IM6/F584	I	Circular	6.35	H-92-A	60.3	490.0	1.017	498.4
IM6/F584	I	Circular	6.35	H-92-B	59.4	483.4	1.017	491.7
IM6/F584	I	Circular	6.35	H-92-D	60.4	483.8	1.017	492.1
T300/5208	I	Circular	6.35	H-13-A	52.9	286.6	1.017	291.5
T300/5208	I	Circular	6.35	H-13-B	54.7	287.8	1.017	292.7
T300/5208	I	Circular	6.35	H-13-C	55.4	271.0	1.017	275.6

* see Table 2

TABLE 5. (CONTINUED)

Material Name	Lay-up*	Hole Type	Diameter mm	Specimen Number	Modulus @ .004 Strain (E_y), GPa	Ultimate Tensile Strength (σ_N), MPa	Finite Width Correction Factor $\left(k_T/k_T^{\infty}\right)$	Infinite Tensile Strength $\left(\sigma_N^{\infty}\right)$, MPa
AS4/3501-6	II	Circular	6.71	232.C01	86.9	499.0	1.035	516.7
AS4/3501-6	II	Circular	6.71	232.C02	82.7	509.3	1.035	527.4
AS4/3501-6	II	Circular	6.71	232.C03	82.7	562.6	1.035	582.5
AS4/3501-6	II	Circular	6.71	232.C04	82.0	502.4	1.035	520.1
AS4/3501-6	II	Circular	6.71	232.C06	84.8	513.5	1.035	531.7
AS4/3501-6	II	Circular	6.71	232.C07	82.0	528.3	1.035	547.0
AS4/3501-6	II	Circular	6.71	232.C08	82.0	501.1	1.035	518.8
AS4/3501-6	II	Circular	6.71	232.C09	84.8	545.5	1.035	564.8
AS4/3501-6	II	Circular	6.71	232.C10	84.8	530.8	1.035	549.6
AS4/3501-6	II	Circular	6.71	232.C11	81.4	510.9	1.035	529.0
AS4/3501-6	II	Circular	6.71	232.C12	79.3	492.8	1.035	510.2

* see Table 2

TABLE 6. RESULTS OF THE DETERMINATION OF THE WEIBULL PARAMETERS FOR UNNOTCHED TEST DATA

Material Name	Lay-Up*	Rank (i)	Probability of Failure $P(\sigma_o)$	Abacissa (X_p)	Ordinate (Y_p)	Summary of Regression Analysis
IM6/5245C	I	1	0.046	-3.068	6.641	
IM6/5245C	I	2	0.136	-1.920	6.668	$Y_p = 6.739 + 0.040\ X_p$
IM6/5245C	I	3	0.227	-1.356	6.669	
IM6/5245C	I	4	0.318	-0.960	6.669	$k_o = 0.040$
IM6/5245C	I	5	0.409	-0.642	6.704	
IM6/5245C	I	6	0.500	-0.367	6.729	$X_p(P(\sigma_o) = 0.625) = -0.019$
IM6/5245C	I	7	0.591	-0.112	6.736	
IM6/5245C	I	8	0.682	0.136	6.741	$Y_p(P(\sigma_o) = 0.625) = 6.738$
IM6/5245C	I	9	0.773	0.393	6.750	
IM6/5245C	I	10	0.864	0.689	6.760	$\beta_o = e^{6.738} = 843.7$ MPa
IM6/5245C	I	11	0.955	1.129	6.816	
AS4/3501-6	II	1	0.042	-3.157	6.984	
AS4/3501-6	II	2	0.125	-2.013	7.018	$Y_p = 7.072 + 0.032\ X_p$
AS4/3501-6	II	3	0.208	-1.454	7.021	
AS4/3501-6	II	4	0.292	-1.065	7.037	$k_o = 0.032$
AS4/3501-6	II	5	0.375	-0.755	7.047	
AS4/3501-6	II	6	0.458	-0.489	7.048	$X_p(P(\sigma_o) = 0.625) = -0.019$
AS4/3501-6	II	7	0.542	-0.248	7.052	
AS4/3501-6	II	8	0.625	-0.019	7.060	$Y_p(P(\sigma_o) = 0.625) = 7.072$
AS4/3501-6	II	9	0.708	0.209	7.065	
AS4/3501-6	II	10	0.792	0.450	7.072	$\beta_o = e^{7.072} = 1178.4$ MPa
AS4/3501-6	II	11	0.875	0.732	7.115	
AS4/3501-6	II	12	0.958	1.156	7.140	

* see Table 2

TABLE 7. RESULTS OF THE DETERMINATION OF THE WEIBULL PARAMETERS FOR NOTCHED TEST DATA
(a) MATERIAL = IM6/5245C

Diameter mm	Lay-Up*	Rank (i)	Probability of Failure $P(\sigma_o)$	Abacissa (X_p)	Ordinate (Y_p)	Summary of Regression Analysis
6.35	I	1	0.046	-3.068	6.188	
6.35	I	2	0.136	-1.920	6.196	$Y_p = 6.234 + 0.020\ X_p$
6.35	I	3	0.227	-1.356	6.200	
6.35	I	4	0.318	-0.960	6.203	$k_N = 0.020$
6.35	I	5	0.409	-0.642	6.221	
6.35	I	6	0.500	-0.367	6.229	$X_p(P(\sigma_o) = 0.625) = -0.019$
6.35	I	7	0.591	-0.112	6.230	
6.35	I	8	0.682	0.136	6.231	$Y_p(P(\sigma_o) = 0.625) = 6.234$
6.35	I	9	0.773	0.393	6.239	
6.35	I	10	0.864	0.689	6.243	$\beta_N^{\infty} = e^{6.234} = 509.8$ MPa
6.35	I	11	0.955	1.129	6.279	
9.53	I	1	0.046	-3.068	6.077	
9.53	I	2	0.136	-1.920	6.079	$Y_p = 6.123 + 0.019\ X_p$
9.53	I	3	0.227	-1.356	6.091	
9.53	I	4	0.318	-0.960	6.103	$k_N = 0.019$
9.53	I	5	0.409	-0.642	6.106	
9.53	I	6	0.500	-0.367	6.111	$X_p(P(\sigma_o) = 0.625) = -0.019$
9.53	I	7	0.591	-0.112	6.127	
9.53	I	8	0.682	0.136	6.128	$Y_p(P(\sigma_o) = 0.625) = 6.123$
9.53	I	9	0.773	0.393	6.129	
9.53	I	10	0.864	0.689	6.130	$\beta_N^{\infty} = e^{6.123} = 456.0$ MPa
9.53	I	11	0.955	1.129	6.155	

* see Table 2

TABLE 7. RESULTS OF THE DETERMINATION OF THE WEIBULL PARAMETERS FOR NOTCHED TEST DATA
(b) MATERIAL = AS4/3501-6

Diameter mm	Lay-Up*	Rank (i)	Probability of Failure $P(\sigma_o)$	Abacissa (X_p)	Ordinate (Y_p)	Summary of Regression Analysis
6.71	II	1	0.046	-3.068	6.235	
6.71	II	2	0.136	-1.920	6.247	$Y_p = 6.300 + 0.030\ X_p$
6.71	II	3	0.227	-1.356	6.252	
6.71	II	4	0.318	-0.960	6.254	$k_N = 0.030$
6.71	II	5	0.409	-0.642	6.268	
6.71	II	6	0.500	-0.367	6.271	$X_p(P(\sigma_o) = 0.625) = -0.019$
6.71	II	7	0.591	-0.112	6.276	
6.71	II	8	0.682	0.136	6.304	$Y_p(P(\sigma_o) = 0.625) = 6.300$
6.71	II	9	0.773	0.393	6.309	
6.71	II	10	0.864	0.689	6.337	$\beta_N^{\infty} = e^{6.300} = 544.7$ MPa
6.71	II	11	0.955	1.129	6.367	

* see Table 2

TABLE 8. CHARACTERISTIC DIMENSIONS FOR IM6/5245C AND AS4/3501-6

Material Name	Lay-Up*	Diameter mm	Characteristic Strength Ratio $\left(\beta_N^{\infty}/\beta_o\right)$	Characteristic Dimension (A_o), mm
IM6/5245C	I	6.35	0.60	3.43
IM6/5245C	I	9.53	0.54	3.43
AS4/3501-6	II	6.71	0.46	1.48

* see Table 2

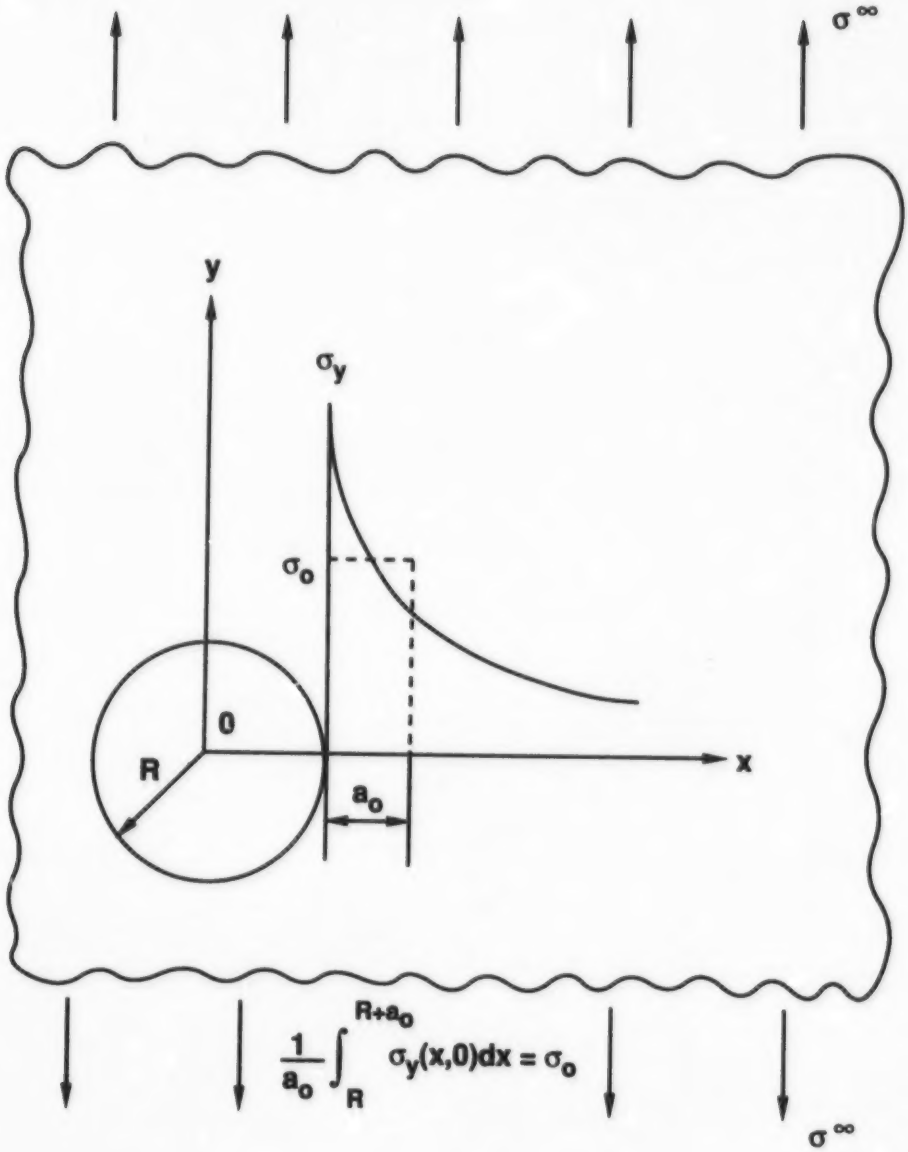

FIG. 1: AVERAGE STRESS FAILURE CRITERION FOR A CIRCULAR THROUGH-THE-THICKNESS HOLE

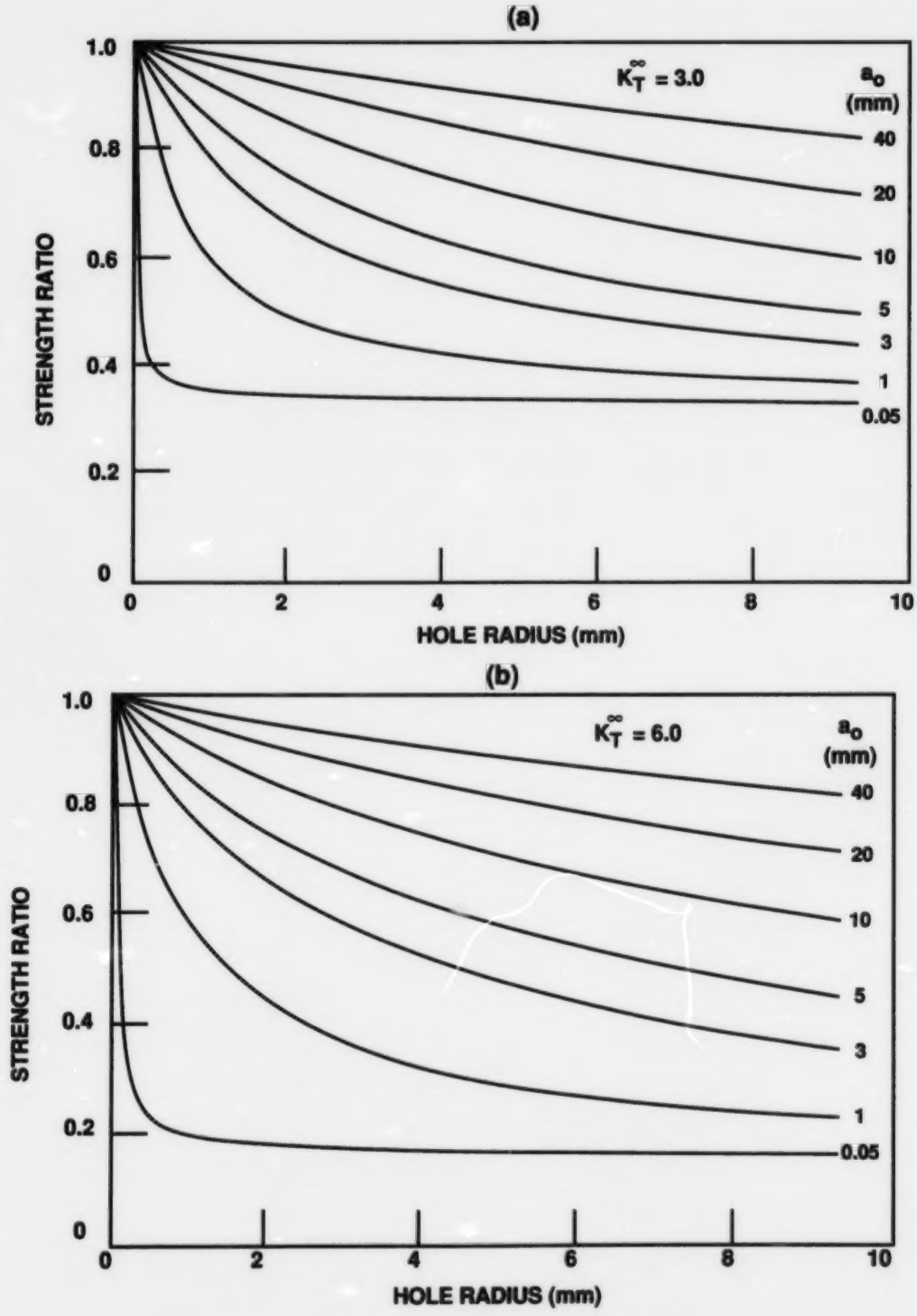

FIG. 2: EFFECT OF THE CHARACTERISTIC DIMENSION, a_o ON NOTCHED STRENGTH PREDICTION FOR LAMINATES CONTAINING A CIRCULAR HOLE HAVING: (a) K_T^∞ = 3.0; (b) K_T^∞ = 6.0

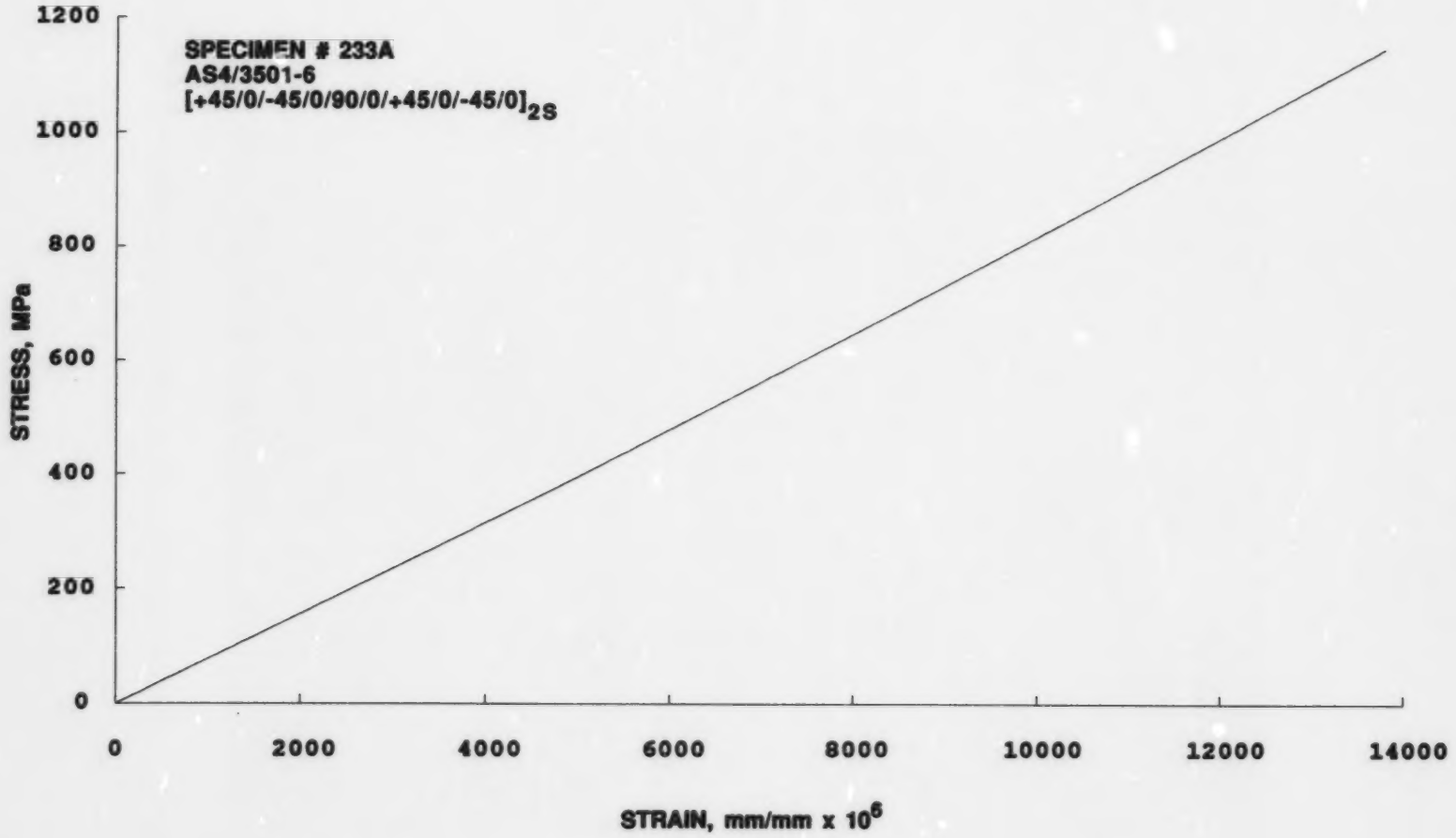

FIG. 3: A TYPICAL STRESS vs. STRAIN PLOT FOR AN UNNOTCHED TENSILE TEST

FIG. 4: A TYPICAL SETUP FOR A NOTCHED TENSILE TEST

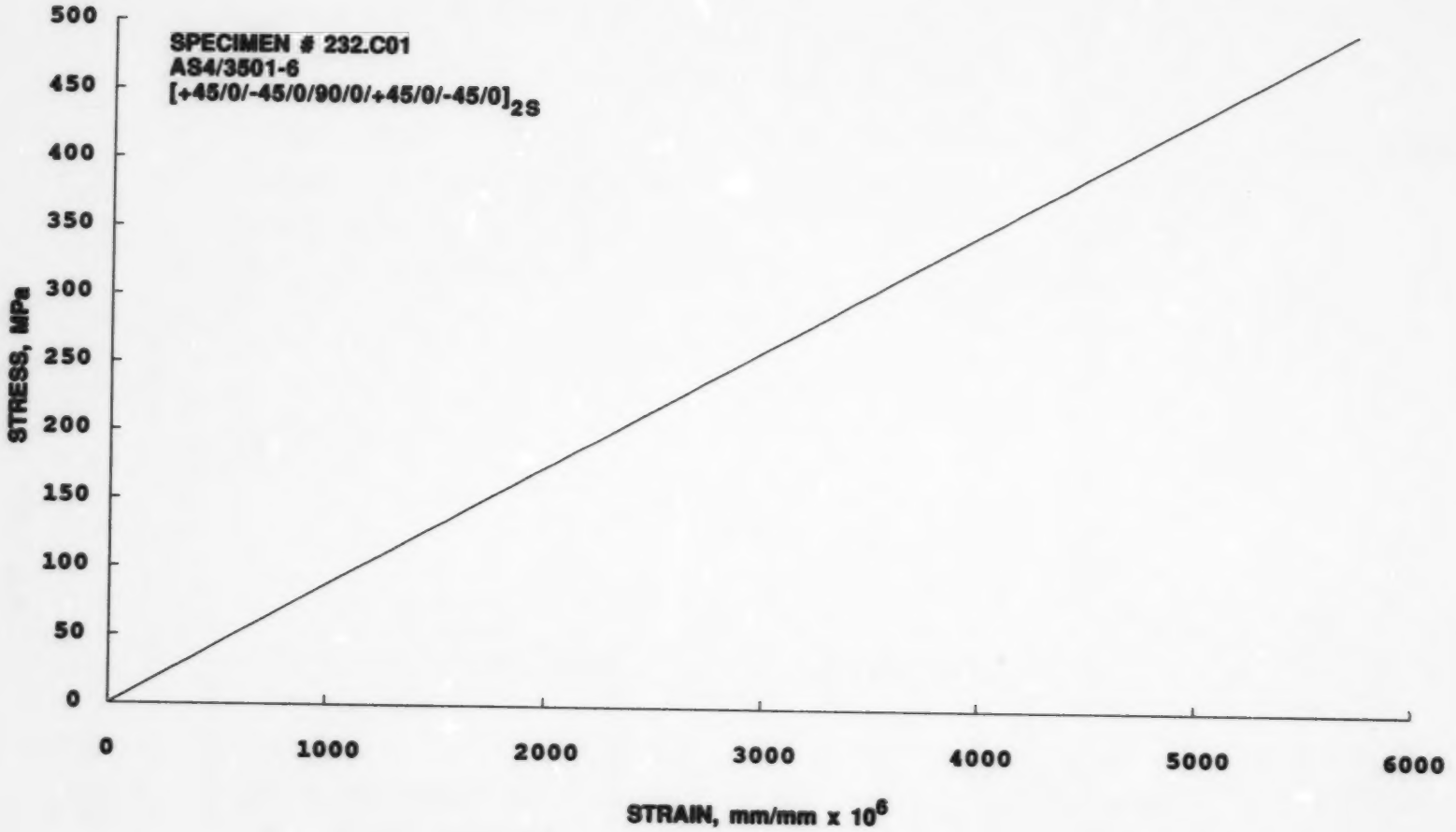

FIG. 5: A TYPICAL STRESS vs. STRAIN PLOT FOR A NOTCHED TENSILE TEST

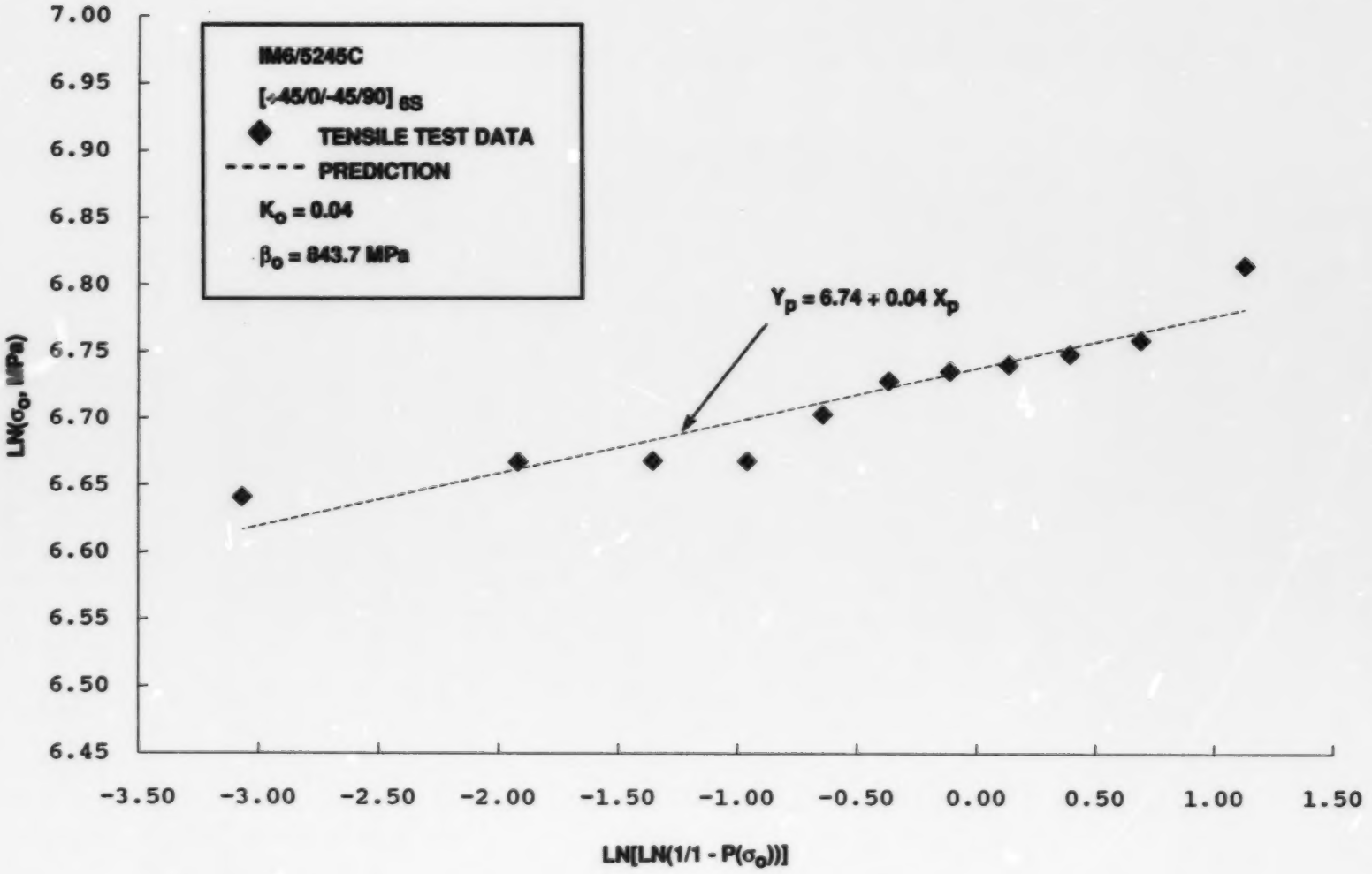

FIG. 6: WEIBULL PROBABILITY PLOT FOR IM6/5245 UNNOTCHED STRENGTH DATA

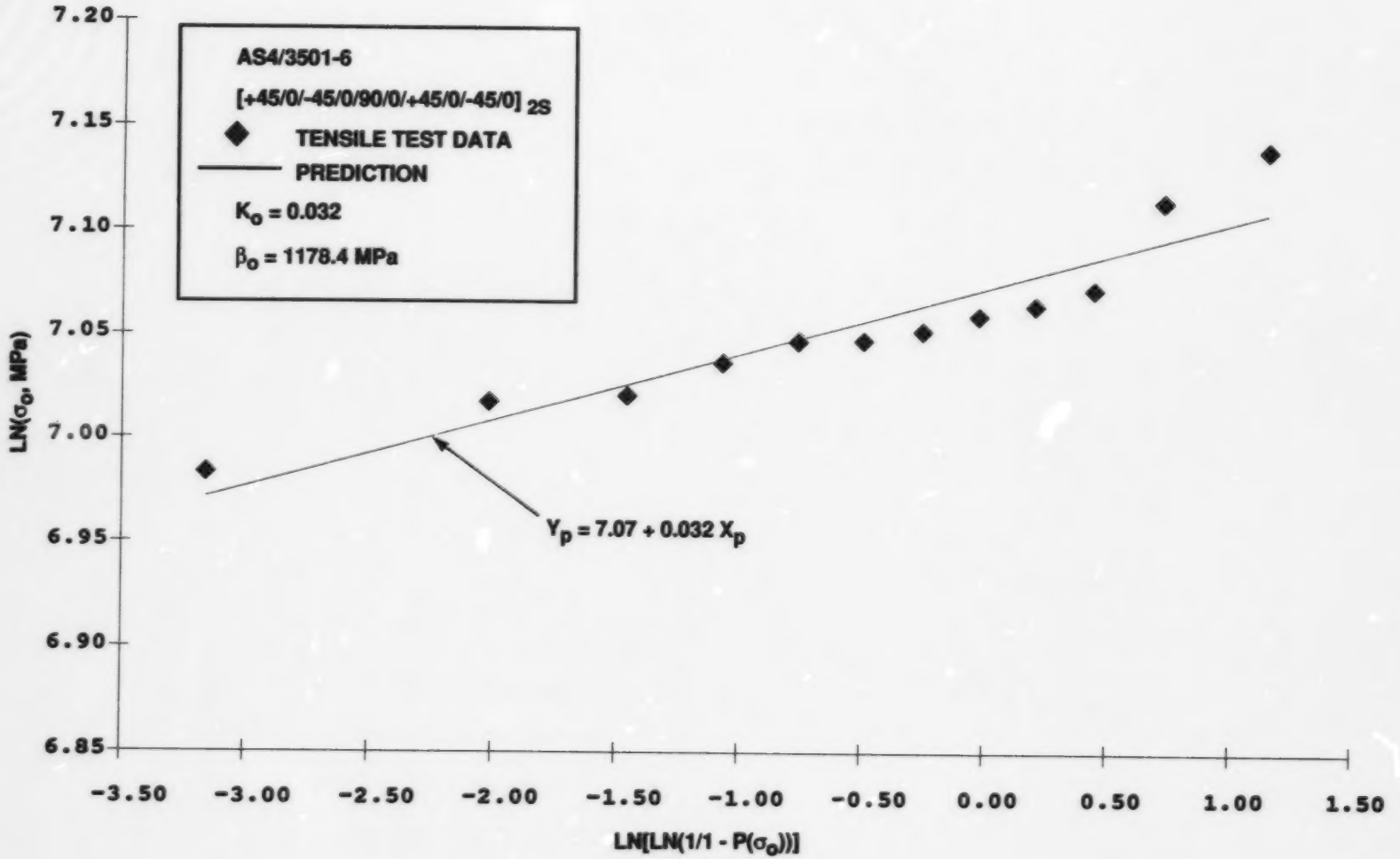

FIG. 7: WEIBULL PROBABILITY PLOT FOR AS4/3501-6 UNNOTCHED STRENGTH DATA

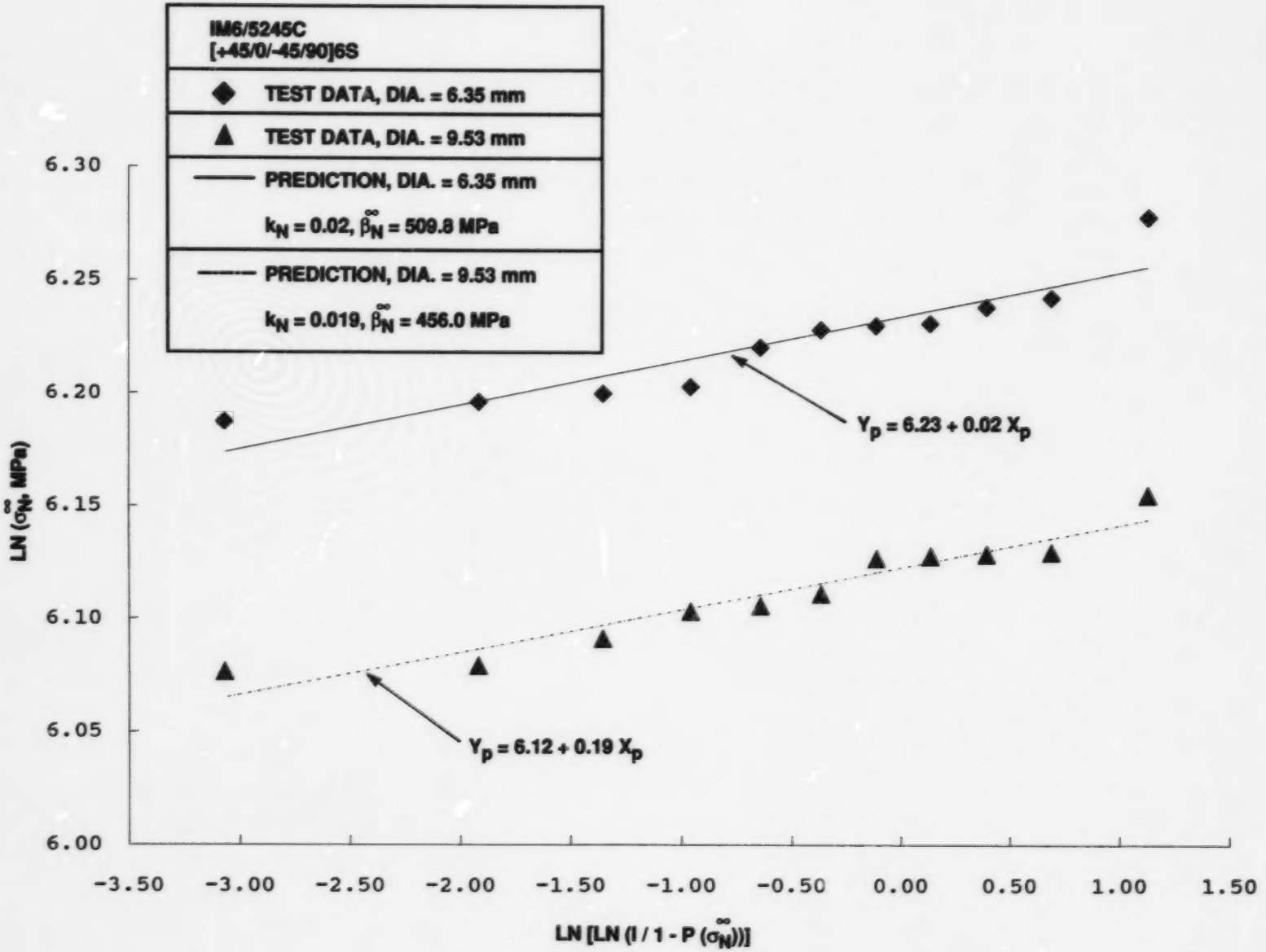

FIG. 8: WEIBULL PROBABILITY PLOT FOR IM6/5245C NOTCHED STRENGTH DATA

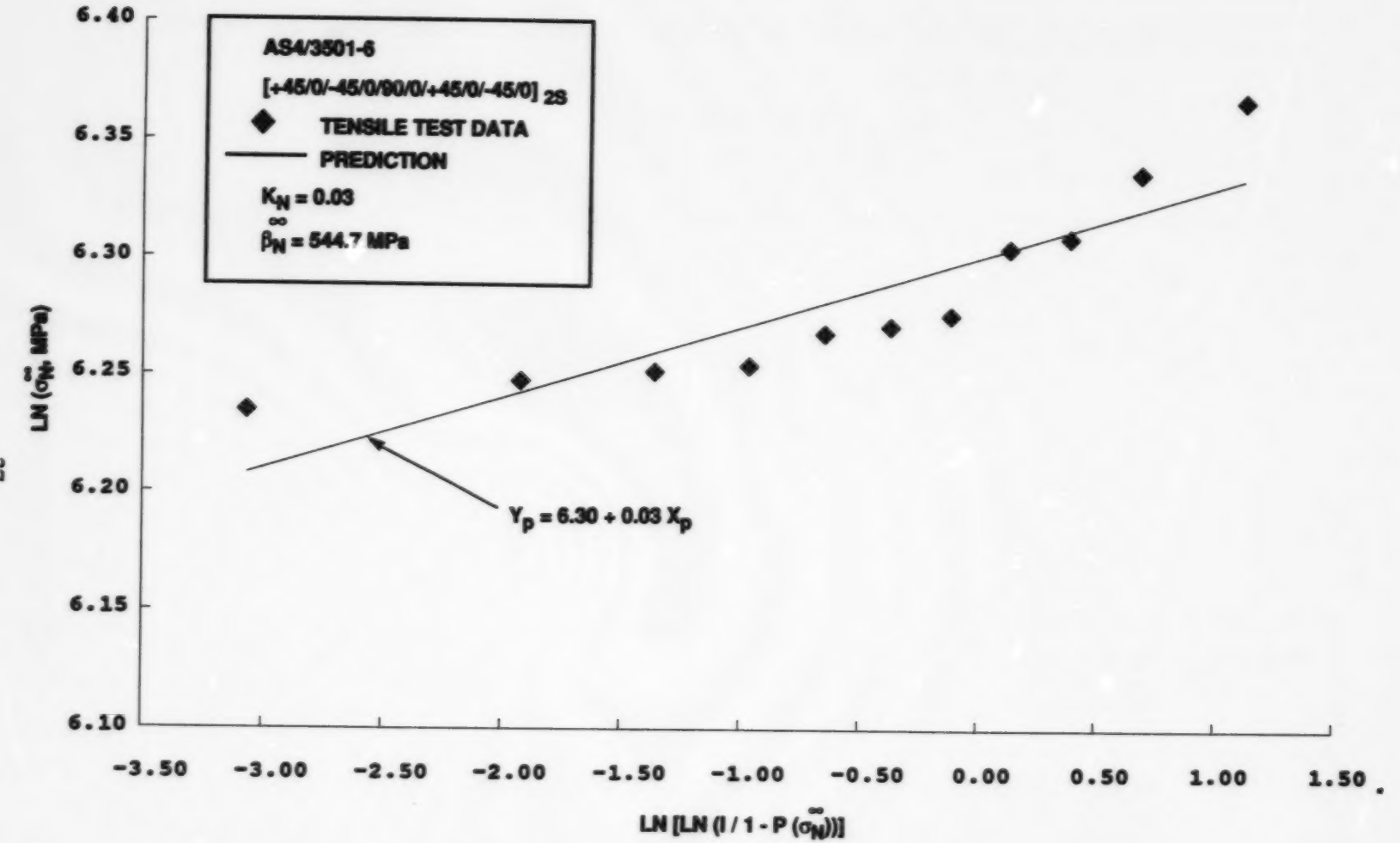

FIG. 9: WEIBULL PROBABILITY PLOT FOR AS4/3501-6 NOTCHED STRENGTH DATA

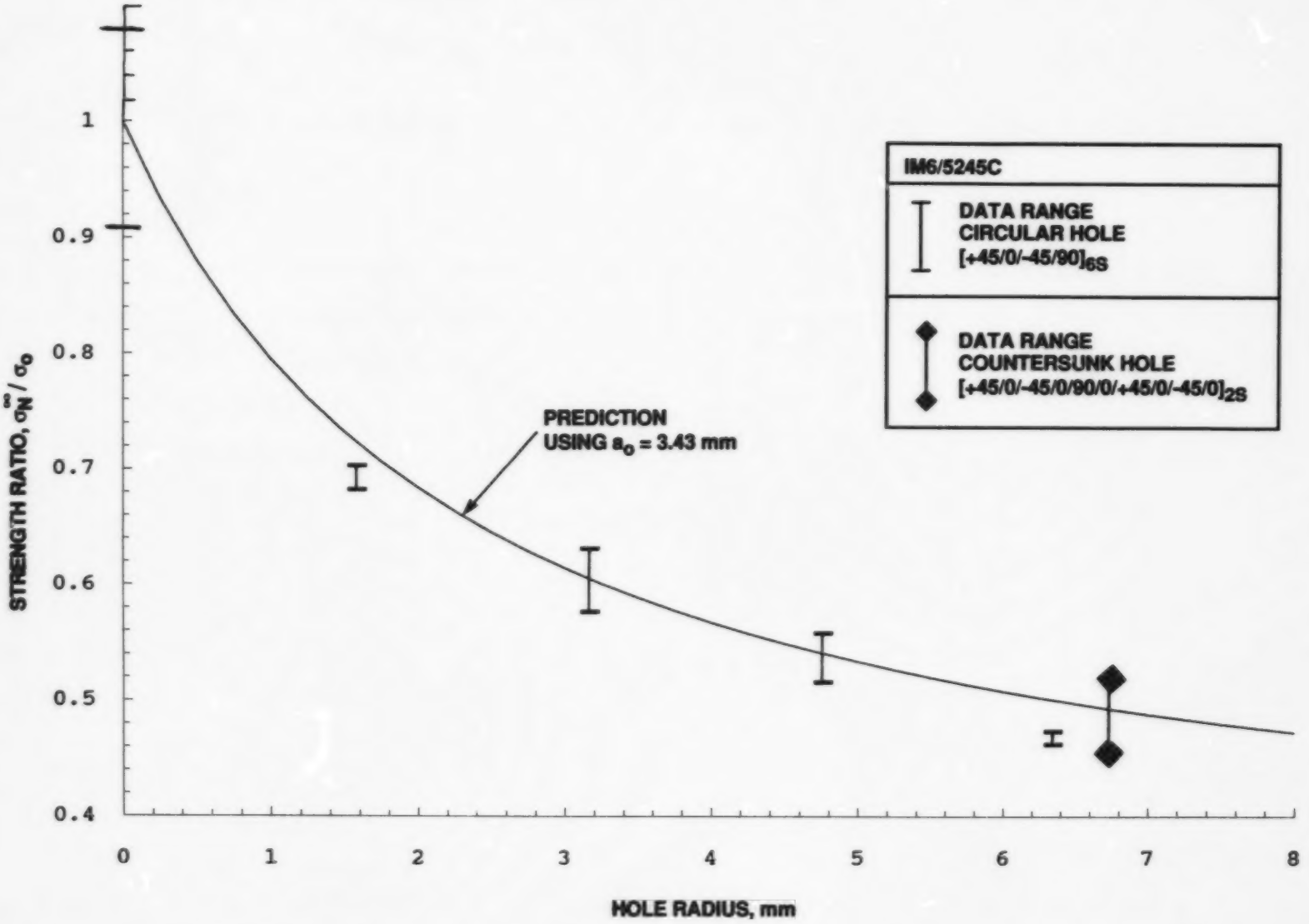

FIG. 10: COMPARISON BETWEEN PREDICTED $\sigma_N^\infty / \sigma_o$ AND TEST RESULTS FOR IM6/5245C

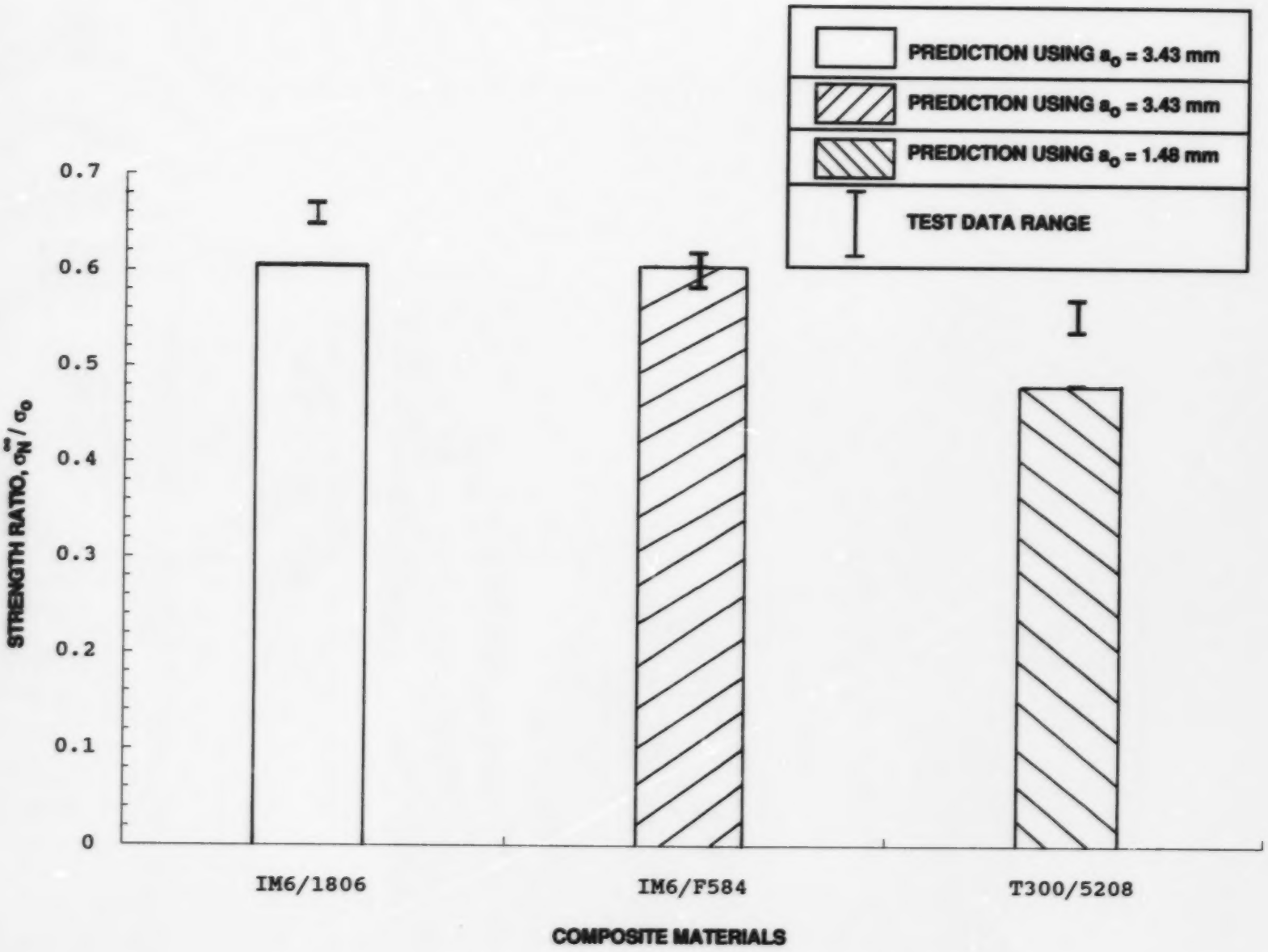

FIG. 11: COMPARISON BETWEEN PREDICTED $\tilde{\sigma}_N / \sigma_o$ AND TEST RESULTS FOR DIFFERENT COMPOSITES

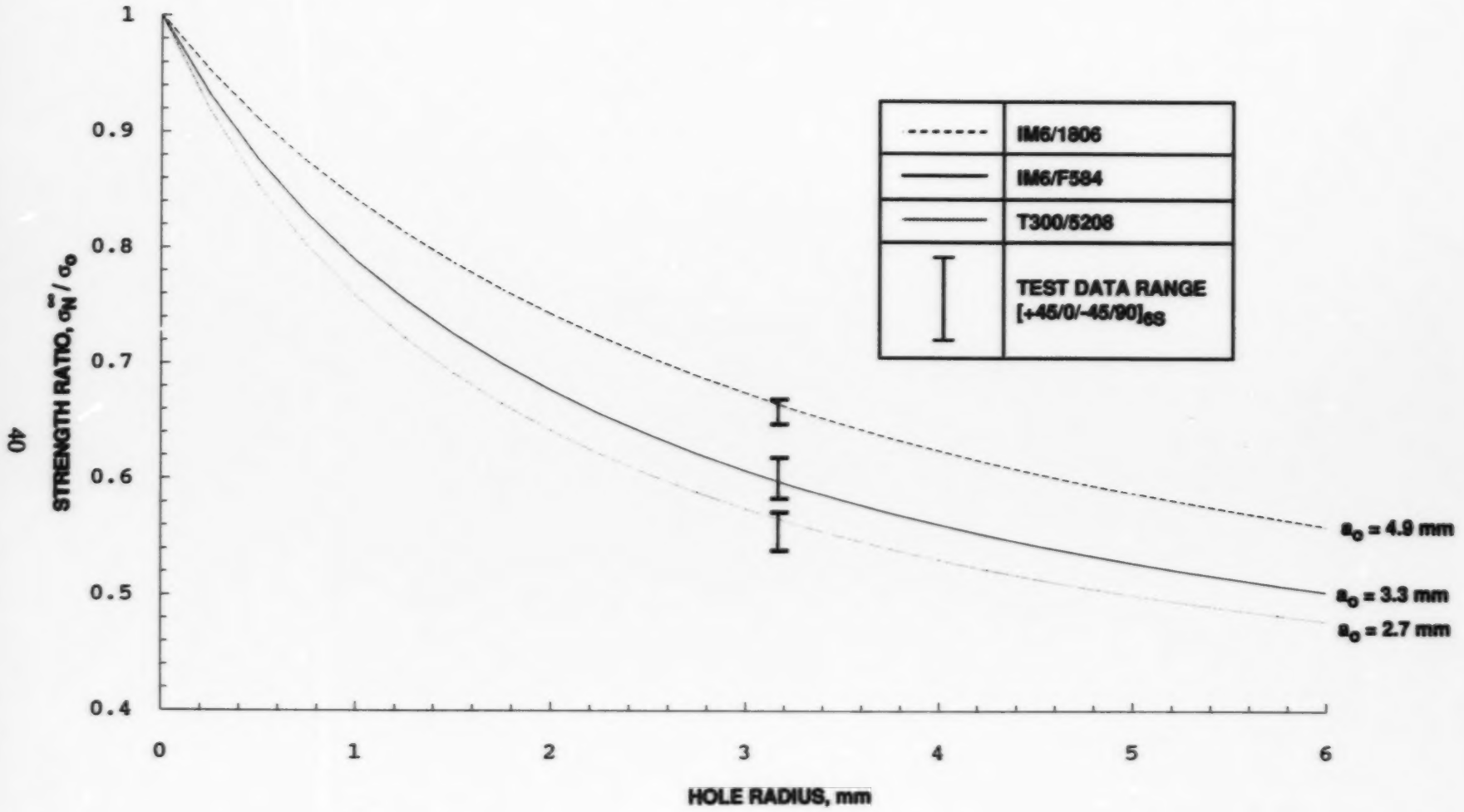

FIG. 12: EFFECTS OF CHARACTERISTIC DIMENSION ON NOTCH SENSITIVITY

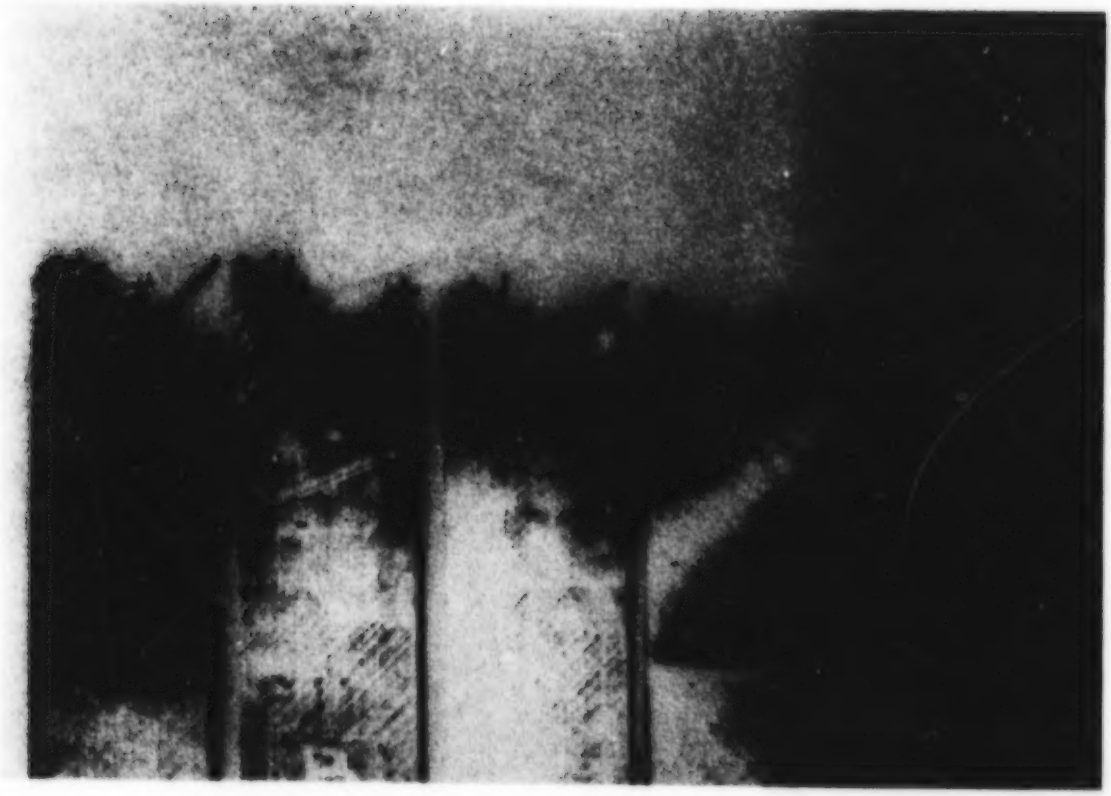

FIG. 13: EFFECTS ON FAILURE MODE RESULTING FROM INCREASING HOLE SIZE

FIG. 14: FLAT FRACTURE OF AN IM6/5245C LAMINATE CONTAINING A COUNTERSUNK HOLE

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DESCRIPTORS (KEY WORDS)/MOTS-CLÉS

14 1. Composite Materials 2. Stress Management 3. Laminates
4. Notch Tests 5. Weibull Density Functions 6. Tensile Strength

SUMMARY/SOMMAIRE

This report describes the determination of empirical parameters for a fracture model which can be used to predict the uniaxial tensile strength of composite laminates containing notches of various sizes. The parameters required are the characteristic dimension and the unnotched tensile strength of the subject laminate. A data base including notched and unnotched strengths of the subject laminates and elastic ply properties was generated for the determination of these parameters using a statistical procedure based on the two parameter Weibull distribution. Three second generation high toughness graphite fibre reinforced composites, IM6/5245C, IM6/F584 AND IM6/1806, and two first generation brittle graphite/epoxy composites, AS4/3501-6 and T300/5208, were used to fabricate the specimens containing either a through-the-thickness cylindrical hole or a countersunk hole.

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